

Realistic Four-Generation MSSM in Type II String Theory

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Abstract

We construct a four-generation MSSM with rank-4 Yukawa matrices from intersecting D6 branes on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. The Yukawa matrices obtained provide an example of Flavor Democracy (FD), where the Yukawa couplings are all nearly equal. Mass hierarchies may then be generated by slight perturbations away from FD. We find that it is possible to obtain hierarchical masses for the quarks and leptons of each generation and mixings between them. In addition, the tree-level gauge couplings are unified at the string scale. Finally, we also construct similar models with one, two, and three generations in which the rank of the Yukawa matrices is equal to the number of generations in each model.

1. Introduction. The main challenge of string phenomenology is to exhibit at least one string vacuum that describes the physics of our universe in every detail. Despite progress in this direction, thus far this goal remains far from achieved. In the past decade, a promising approach to model building has emerged involving compactifications with D branes on orientifolds (for reviews, see [1, 2, 3, 4]). In such models chiral fermions—an intrinsic feature of the Standard Model (SM)—arise from configurations with D branes located at transversal orbifold/conifold singularities [5] and strings stretching between D branes intersecting at angles [6, 7] (or, in its T-dual picture, with magnetized D branes [8, 9, 10]). A number of non-supersymmetric intersecting D-brane models have been constructed that strongly resemble the SM.

However, non-supersymmetric low-energy limits of string theory suffer from internal inconsistencies of noncanceled NS-NS tadpoles, yielding models that destabilize the hierarchy of scales [11]. A resolution of these issues necessarily requires $\mathcal{N} = 1$ supersymmetry. The first semirealistic models that preserve the latter were built in Type IIA theory on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold [12, 13]. Subsequently, intersecting D-brane models based on SM-like, Pati-Salam [14], and $SU(5)$ [15] gauge groups were constructed within the same framework and systematically studied in Refs. [16, 17, 18, 19]. The statistics of 3- and 4-generation models was studied in [20, 21]. Phenomenologically interesting models have also been constructed on a $T^6/(\mathbb{Z}_6)$ orientifold [22]. In addition, several different models with flipped $SU(5)$ [23] have been suggested within intersecting D-brane scenarios [24, 25], as well as models with interesting discrete-group flavor structures [26].

Although F-theory model building [27] has received a great deal of attention lately, intersecting D-brane models remain of great interest. While F theory is useful for overcoming the problem of phenomenologically necessary but perturbatively forbidden Yukawa couplings, one can also evade this problem in intersecting D-brane models by utilizing the Pati-Salam gauge group $SU(4) \times SU(2)_L \times SU(2)_R$ [16], or by incorporating nonperturbative effects such as D-brane instantons [28, 29, 30]. Furthermore, the constraints that must be satisfied to construct globally consistent intersecting D-brane models are well understood (see, *e.g.*, [3, 31]). Thus, intersecting D-brane constructions offer an exciting avenue for model-building, particularly for building SM-like and left-right symmetric models based on the Pati-Salam gauge group.

The SM exhibits an intricate pattern of mass hierarchies and mixings between the different generations. One challenge of any string construction is to explain this structure. Within the framework of D-brane modeling it was demonstrated that the Yukawa matrices $Y_{abc} \sim \exp(-A_{abc})$ arise from worldsheet areas A_{abc} spanning D branes (labeled by a, b, c) supporting fermions and Higgses at their intersections [7, 32]. This pattern naturally encodes the hierarchy of Yukawa couplings. However, for most string constructions, Yukawa matrices are of rank one. In the case of D-brane models built on toroidal orientifolds, this result can be traced to the fact that not all of the intersections at which the SM fermions are localized occur on the same torus. To date only one three-generation model is known in which this problem has been overcome [33], and for which one can obtain mass matrices for quarks and leptons that nearly reproduce experimental values. Additionally, this model exhibits automatic gauge coupling unification at the string scale, and all extra matter can be decoupled. It should be commented that the rank-1 problem for toroidal models can also potentially be solved by D-brane instantons [28, 34, 35]. However, the conditions for including these nonperturbative effects are very constraining, and at present there are no concrete realizations in the literature in which all constraints may be satisfied.

Although present high-energy experimental data supports just three generations of chiral fermions, a fourth generation remains viable as long as the mass of the extra neutrino ν' is larger than $\frac{1}{2}M_Z$, and the fourth-generation charged-fermion masses $m_{t'}$, $m_{b'}$, and $m_{\tau'}$ lie in the

correct mass ranges to avoid constraints from direct searches and precision electroweak measurements [36]. Recent WMAP7 [37] analysis points towards a higher (than three) number of relativistic neutrino species, $N_{\text{eff}} = 4.34^{+0.86}_{-0.88}$, and explanations have been proposed based on sterile neutrinos with masses at sub-eV scales [38] (see also Refs. [39] for earlier discussion of astrophysical implications of the fourth generation). Furthermore, the existence of a fourth generation can introduce additional CP-violating effects [40, 41, 42, 43] and can have a strong effect on Higgs boson searches at colliders [44]. While not directly related to our present focus, these considerations suggest possible manifestations of a fourth generation of fermions.

Interestingly, the principle of Flavor Democracy (FD) [45], where the Yukawa couplings for quarks and leptons are all nearly equal, appears to favor the existence of four generations of chiral fermions. In this scenario, the observed mass hierarchies are generated as a by-product of slight variations away from FD, which may result from the internal geometry of the string construction. For example, the down-type quark masses can be generated naturally in the FD approach by a Yukawa matrix which is nearly rank 1, while up-type quark masses can be generated by a Yukawa matrix which is nearly rank 2, corresponding to near degeneracies in the Yukawa couplings. Furthermore, in the FD approach a seesaw mechanism is not necessary to obtain small neutrino masses, and the scenario of three light neutrinos with a heavy fourth is naturally obtained by considering small perturbations away from FD such that the Yukawa matrix for neutrinos is nearly rank 1. This construction is desirable in intersecting D-brane models since a Majorana mass term for right-handed neutrinos is perturbatively forbidden and can only be obtained nonperturbatively with D-brane instantons. In fact, all Yukawa couplings for quarks and leptons can be allowed perturbatively in intersecting D-brane models with a Pati-Salam gauge group, and in such models a standard Majorana term of the form $W_M = MNN$ may not even be generated through D-brane instantons since it is forbidden by $U(1)_{B-L}$, which is gauged at the string scale. If light, sterile neutrinos turn out to exist, it may be an indication that nature does not utilize the seesaw mechanism to achieve tiny neutrino masses. Finally, FD may reduce the amount of fine-tuning necessary to obtain hierarchical Yukawa couplings in these models since it is natural for the Yukawa couplings to be nearly degenerate, with the observed mass hierarchies being generated by slight departures from this degeneracy.

In the following, we develop a four-generation model constructed from intersecting D6 branes on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold in which one can obtain realistic mass matrices. In particular, the Yukawa matrices are rank 4, and one can obtain nontrivial masses for each generation, with the mass spectrum over the generations being naturally hierarchical. Additionally, the MSSM gauge couplings are unified at the string scale. Also, the hidden-sector gauge groups introduced to satisfy the Ramond-Ramond (RR) tadpole cancellation conditions become confining at high energies. Finally, we construct similar models with one, two, and three generations of matter in which the rank of the Yukawa matrices equals the number of generations.

2. Rank-Four MSSM. The configuration of D branes must obey a number of conditions in order to be a consistent model of particle physics. First, the RR tadpoles vanish via the Gauss' law cancellation condition for the sum of D-brane and cross-cap RR-charges [3, 46]:

$$\sum_{\alpha \in \text{stacks}} N_{\alpha}(\pi_{\alpha} + \pi_{\alpha^*}) - 4\pi_{O6} = 0, \quad (1)$$

written in terms of the three-cycles $\pi_{\alpha} = (n_1^{\alpha}, l_1^{\alpha}) \times (n_2^{\alpha}, l_2^{\alpha}) \times (n_3^{\alpha}, 2^{-\beta} l_3^{\alpha})$ that wrap $(n_j^{\alpha}, m_j^{\alpha})$ times the fundamental cycles $([a_j], [b_j])$ of the factorizable six-torus $T^6 = \prod_{j=1}^3 T_{(j)}^2$. Here, the first two

Table 1: General spectrum for intersecting D6 branes at generic angles, where $I_{aa'} = -2^{3-\beta} \prod_{i=1}^3 (n_a^i l_a^i)$, and $I_{aO6} = 2^{3-\beta} (-l_a^1 l_a^2 l_a^3 + l_a^1 n_a^2 n_a^3 + n_a^1 l_a^2 n_a^3 + n_a^1 n_a^2 l_a^3)$. \mathcal{M} is the multiplicity, and a_S and a_A denote the symmetric and antisymmetric representations of $U(N_a/2)$, respectively.

Sector	Representation
aa	$U(N_a/2)$ vector multiplet and 3 adjoint chiral multiplets
$ab + ba$	$\mathcal{M}(\frac{N_a}{2}, \frac{N_b}{2}) = I_{ab} = 2^{-\beta} \prod_{i=1}^3 (n_a^i l_b^i - n_b^i l_a^i)$
$ab' + b'a$	$\mathcal{M}(\frac{N_a}{2}, \frac{N_b}{2}) = I_{ab'} = -2^{-\beta} \prod_{i=1}^3 (n_a^i l_b^i + n_b^i l_a^i)$
$aa' + a'a$	$\mathcal{M}(a_S) = \frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6})$; $\mathcal{M}(a_A) = \frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO6})$

two-tori are rectangular: $l_j^\alpha = m_j^\alpha$ ($j = 1, 2$), while the third two-torus can be rectangular ($\beta=0$), or tilted such that $l_3^\alpha = 2m_j^\alpha + n_j^\alpha$ and $\beta = 1$. In the T-dual picture the tilt of the third cycle $[a'_3] = [a_3] + \frac{1}{2}[b_3]$ corresponds to turning on a non-zero NS-NS two-form B field. However, it becomes nondynamical under the requirement of its invariance under the orientifold projection $\Omega\mathcal{R}$ [47]. As a consequence, its flux can admit only two discrete values, resulting in two discrete values for β . Each two-torus possesses the complex structure modulus $\chi_j = R_2^{(j)}/R_1^{(j)}$ built from its radii $R_1^{(j)}$ and $R_2^{(j)}$. $\mathcal{N} = 1$ supersymmetry, which is favored for reasons of underlying consistent low-energy theories of particle physics as well as for stability of D-brane configurations, is preserved by choosing the angles between the D-brane stacks and orientifold planes to obey the condition [12, 13]

$$\theta_1^\alpha + \theta_2^\alpha + \theta_3^\alpha = 0 \bmod 2\pi, \quad (2)$$

with $\theta_j^\alpha = \arctan(2^{-\beta_j} \chi_j l_j^\alpha / n_j^\alpha)$ and $\beta_{1,2} = 0$ and $\beta_3 = \beta$. This condition can be written in terms of wrapping numbers satisfying the two equations

$$\begin{aligned} x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a &= 0, \\ A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D &< 0, \end{aligned} \quad (3)$$

where

$$\begin{aligned} \tilde{A}_a &= -l_a^1 l_a^2 l_a^3, & \tilde{B}_a &= l_a^1 n_a^2 n_a^3, & \tilde{C}_a &= n_a^1 l_a^2 n_a^3, & \tilde{D}_a &= n_a^1 n_a^2 l_a^3, \\ A_a &= -n_a^1 n_a^2 n_a^3, & B_a &= n_a^1 l_a^2 l_a^3, & C_a &= l_a^1 n_a^2 l_a^3, & D_a &= l_a^1 l_a^2 n_a^3, \end{aligned} \quad (4)$$

and x_A , x_B , x_C , and x_D are the complex structure parameters [17], where $x_A = \lambda$, $x_B = \lambda \cdot 2^{\beta_2+\beta_3}/\chi_2\chi_3$, $x_C = \lambda \cdot 2^{\beta_1+\beta_3}/\chi_1\chi_3$, $x_D = \lambda \cdot 2^{\beta_1+\beta_2}/\chi_1\chi_2$, and λ is a positive parameter that puts the parameters A , B , C , and D on equal footing. Furthermore, the consistency of the model is further ensured by the K-theory conditions [31, 48], which imply the cancellation of the Z_2 charges carried by D branes in orientifold compactifications in addition to the vanishing of the total homological charge exhibited by Eq. (1). In the present case, nonvanishing torsion charges are avoided by considering stacks with an even number of D branes, *i.e.*, $N_\alpha \in 2\mathbb{Z}$.

Imposing these constraints, we present the D6-brane configurations, intersection numbers, and complex structure parameters of the model in Table 2, and the resulting spectrum in Table 3, with formulas for calculating the multiplicity of states in bifundamental, symmetric, and

antisymmetric states shown in Table 1. Models with different numbers of generations may be obtained for different values of the wrapping number n_g as well as the third-torus tilt parameter β . The observable sector of the models then has the gauge symmetry and matter content of an $(N_g = 2^{1-\beta}n_g)$ -generation SM with an extended Higgs sector. The extra matter in the models consists of matter charged under the hidden-sector gauge groups, and vectorlike matter between pairs of branes that do not intersect, as well as the chiral adjoints associated with each stack of branes. In addition, one has matter in the symmetric triplet representation of $SU(2)_L$ as well as additional singlets. In order to have just the MSSM at low energies, the gauge couplings must unify at some energy scale, and all extra matter besides the MSSM states must become massive at high-energy scales. Furthermore, one requires just one pair of Higgs doublets.

The resulting models have gauge symmetry $[U(4)_C \times U(2)_L \times U(2)_R]_{\text{observable}} \times [\text{USp}(2^{2-\beta}(4 - n_g))^2 \times \text{USp}(2^{2-\beta})^2]_{\text{hidden}}$. The hidden sector, as well as the set of complex structure parameters required to preserve $\mathcal{N} = 1$ supersymmetry, is different in each of the models with different numbers of generations. In particular, in the tilted case two of the hidden-sector gauge groups fall out in going from three-generation to four-generation models. The non-Abelian chiral anomalies vanish as a consequence of the RR tadpole condition (1). The chiral anomalies from the three global $U(1)$ s of $U(4)_C$, $U(2)_L$, and $U(2)_R$ inducing couplings of the form $A_\alpha \wedge F_\beta^2$, with A and F referring to Abelian and non-Abelian gauge fields, respectively, read [49]

$$\mathcal{A}^{\text{chiral}} = \frac{1}{2} \sum_{\alpha, \beta} N_\alpha (I_{\alpha\beta} - I_{\alpha^*\beta}) A_\alpha \wedge F_\beta^2, \quad (5)$$

However, these anomalies cancel against the couplings induced by RR fields via the Green-Schwarz mechanism [49]:

$$\mathcal{A}^{\text{RR}} = 8n_g A_a \wedge (F_c^2 - F_b^2) + 4n_g A_b \wedge F_a^2 - 4n_g A_c \wedge F_a^2, \quad (6)$$

such that $\mathcal{A}^{\text{chiral}} + \mathcal{A}^{\text{RR}} = 0$. The gauge fields A_α of these $U(1)$ s receive masses via linear $\sum_\ell c_\ell^\alpha B_2^\ell \wedge A_\alpha$ couplings in the ten-dimensional action, with the massless modes given by $\ker(c_\ell^\alpha)$. The latter is trivial in the present model, which means that the effective gauge symmetry of the observable sector is $SU(4)_C \times SU(2)_L \times SU(2)_R$.

In order to break the gauge symmetry of the observable sector down to the SM, we split the a stack of D6 branes on the first two-torus into stacks a_1 and a_2 with $N_{a_1} = 6$ and $N_{a_2} = 2$ D6 branes, and similarly split the c stack of D6 branes into stacks c_1 and c_2 such that $N_{c_1} = 2$ and $N_{c_2} = 2$. The process of brane-splitting corresponds to giving a vacuum expectation value (VEV) to the chiral adjoint fields associated with each stack, which are open-string moduli. The gauge symmetry subsequently breaks down to $SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$, where the $U(1)_{I_{3R}}$ and $U(1)_{B-L}$ gauge bosons remain massless. The $U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry may then be broken to $U(1)_Y = \frac{1}{2}U(1)_{B-L} + U(1)_{I_{3R}}$ by giving VEVs to the vectorlike particles with the quantum numbers $(\mathbf{1}, \mathbf{1}, 1/2, -1)$ and $(\mathbf{1}, \mathbf{1}, -1/2, 1)$ under the $SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$ gauge symmetry arising from $a_2 c_1'$ intersections. The full gauge symmetry of the models is then $SU(3)_C \times SU(2)_L \times U(1)_Y \times [\text{USp}[2^{2-\beta}(4 - n_g)]^2 \times \text{USp}(2^{2-\beta})^2]$, with the hypercharge given by

$$Q_Y = \frac{1}{6} (Q_{a_1} - 3Q_{a_2} - 3Q_{c_1} + 3Q_{c_2}), \quad (7)$$

where the a -stack charges provide Q_{B-L} and the c -stack charges provide Q_{3R} .

The gauge coupling constant associated with a stack α is given by

$$g_{\text{D6}_\alpha}^{-2} = |\Re(f_\alpha)|, \quad (8)$$

Table 2: D6-brane configurations and intersection numbers for a series of Pati-Salam models with $2^{1-\beta}n_g$ generations on a Type IIA $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, where the tadpole conditions are satisfied without introducing fluxes. The parameter β can be zero or one if the third torus is untilted or tilted respectively, while the wrapping number n_g may take the values 1, 2, 3, or 4. The complete gauge symmetry is $[\text{U}(4)_C \times \text{U}(2)_L \times \text{U}(2)_R]_{\text{observable}} \times \{\text{USp}[2^{2-\beta}(4-n_g)]^2 \times \text{USp}(2^{2-\beta})^2\}_{\text{hidden}}$, and the complex structure parameters that preserve $\mathcal{N} = 1$ supersymmetry are $x_A = x_B = n_g \cdot x_C = n_g \cdot x_D$. The parameters β_i^g give the β -functions for the hidden-sector gauge groups.

$\text{U}(4)_C \times \text{U}(2)_L \times \text{U}(2)_R \times \text{USp}[2^{2-\beta}(4-n_g)]^2 \times \text{USp}(2^{2-\beta})^2$												
	N	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$	n_S	n_A	b	b'	c	c'	1	2	3	4
a	8	$(0, -1) \times (1, 1) \times (1, 1)$	0	0	$2^{1-\beta}n_g$	0	$-2^{1-\beta}n_g$	0	1	-1	0	0
b	4	$(n_g, 1) \times (1, 0) \times (1, -1)$	$2^{1-\beta}(n_g - 1)$	$-2^{1-\beta}(n_g - 1)$	-	0	0	0	0	1	0	$-n_g$
c	4	$(n_g, -1) \times (0, 1) \times (1, -1)$	$-2^{1-\beta}(n_g - 1)$	$2^{1-\beta}(n_g - 1)$	-	-	-	0	-1	0	n_g	0
1	$2^{2-\beta}(4-n_g)$	$(1, 0) \times (1, 0) \times (2^\beta, 0)$	$x_A = x_B = n_g \cdot x_C = n_g \cdot x_D \leftrightarrow \chi_1 = n_g, \chi_2 = 1, \chi_3 = 2^\beta$ $\beta_1^g = -3, \beta_2^g = -3$ $\beta_3^g = -6 + n_g$ $\beta_4^g = -6 + n_g$									
2	$2^{2-\beta}(4-n_g)$	$(1, 0) \times (0, -1) \times (0, 2^\beta)$										
3	$2^{2-\beta}$	$(0, -1) \times (1, 0) \times (0, 2^\beta)$										
4	$2^{2-\beta}$	$(0, -1) \times (0, 1) \times (2^\beta, 0)$										

where f_α is the holomorphic gauge kinetic function associated with stack α , given [3, 11] in terms of NS-NS fields by:

$$f_\alpha = \frac{1}{4\kappa_\alpha} [n_1^\alpha n_2^\alpha n_3^\alpha s - 2^{-\beta} n_1^\alpha l_2^\alpha l_3^\alpha u^1 - 2^{-\beta} n_2^\alpha l_1^\alpha l_3^\alpha u^2 - n_3^\alpha l_1^\alpha l_2^\alpha u^3], \quad (9)$$

where $\kappa_\alpha = 1$ for $\text{SU}(N_\alpha)$ and $\kappa_\alpha = 2$ for $\text{USp}(2N_\alpha)$ or $\text{SO}(2N_\alpha)$ gauge groups. The holomorphic gauge kinetic function associated with SM hypercharge $\text{U}(1)_Y$ is then given by taking a linear combination of the holomorphic kinetic gauge functions from all of the stacks [50]:

$$f_Y = \frac{1}{6}f_{a_1} + \frac{1}{2}(f_{a_2} + f_{c_1} + f_{c_2}). \quad (10)$$

Note that in Eq. (9), the four-dimensional dilaton s and complex structure moduli u^i refer to the supergravity basis. These moduli must be stabilized, and gaugino condensation of the effective Veneziano-Yankielowicz Lagrangian [51] provides an example of such a mechanism [52]. Gaugino condensation in the hidden sectors can play an important role in moduli stabilization, and it might provide a top-down reason why three generations is preferred over four.

From the complex structure parameters, the complex structures U^i are determined to be

$$U^1 = n_g \cdot i, \quad U^2 = i, \quad U^3 = -\beta + i. \quad (11)$$

The dilaton and complex structure moduli are then given in the supergravity basis by¹

$$\begin{aligned} \text{Re}(s) &= \frac{1}{(2^\beta n_g)^{1/2}} \frac{e^{-\phi_4}}{2\pi}, & \text{Re}(u^1) &= \left(\frac{2^\beta}{n_g}\right)^{1/2} \frac{e^{-\phi_4}}{2\pi}, \\ \text{Re}(u^2) &= (2^\beta n_g)^{1/2} \frac{e^{-\phi_4}}{2\pi}, & \text{Re}(u^3) &= \frac{1}{(2^\beta n_g)^{1/2}} \frac{e^{-\phi_4}}{2\pi}, \end{aligned} \quad (12)$$

¹See, e.g., footnote 5 of Ref. [50] for the relation between these and complex structures U^i .

Table 3: The chiral and vectorlike superfields of the model, and their quantum numbers under the gauge symmetry $U(4)_C \times U(2)_L \times U(2)_R \times \text{USp}[2^{2-\beta}(4 - n_g)]^2 \times \text{USp}(2^{2-\beta})^2$.

	Multiplicity	Quantum Number	Q_4	Q_{2L}	Q_{2R}	Field
ab	$2^{1-\beta}n_g$	$(4, \bar{2}, 1, 1, 1, 1, 1)$	1	-1	0	$F_L(Q_L, L_L)$
ac	$2^{1-\beta}n_g$	$(\bar{4}, 1, 2, 1, 1, 1, 1)$	-1	0	1	$F_R(Q_R, L_R)$
$a1$	1	$(4, 1, 1, \bar{N}_1, 1, 1, 1)$	1	0	0	X_{a1}
$a2$	1	$(\bar{4}, 1, 1, 1, N_2, 1, 1)$	-1	0	0	X_{a2}
$b2$	1	$(1, 2, 1, 1, \bar{N}_2, 1, 1)$	0	1	0	X_{b2}
$b4$	n_g	$(1, \bar{2}, 1, 1, 1, 1, N_4)$	0	-1	0	X_{b4}^i
$c1$	1	$(1, 1, \bar{2}, N_1, 1, 1, 1)$	0	0	-1	X_{c1}
$c3$	n_g	$(1, 1, 2, 1, 1, \bar{N}_3, 1)$	0	0	1	X_{c3}^i
b_S	$2^{1-\beta}(n_g - 1)$	$(1, 3, 1, 1, 1, 1, 1)$	0	2	0	T_L^i
b_A	$2^{1-\beta}(n_g - 1)$	$(1, \bar{1}, 1, 1, 1, 1, 1)$	0	-2	0	S_L^i
c_S	$2^{1-\beta}(n_g - 1)$	$(1, 1, \bar{3}, 1, 1, 1, 1)$	0	0	-2	T_R^i
c_A	$2^{1-\beta}(n_g - 1)$	$(1, 1, 1, 1, 1, 1, 1)$	0	0	2	S_R^i
ab'	n_g	$(4, 2, 1, 1, 1, 1, 1)$	1	1	0	
	n_g	$(\bar{4}, \bar{2}, 1, 1, 1, 1, 1)$	-1	-1	0	
ac'	n_g	$(4, 1, 2, 1, 1, 1, 1)$	1	0	1	Φ_i
	n_g	$(\bar{4}, 1, \bar{2}, 1, 1, 1, 1)$	-1	0	-1	$\bar{\Phi}_i$
bc	$2n_g$	$(1, 2, \bar{2}, 1, 1, 1, 1)$	0	1	-1	H_u^i, H_d^i
	$2n_g$	$(1, \bar{2}, 2, 1, 1, 1, 1)$	0	-1	1	

where $\phi_4 = \ln g_s$ is the four-dimensional dilaton. Inserting these expressions into Eq. (8), one finds that the gauge couplings are unified as $g_s^2 = g_w^2 = \frac{5}{3}g_Y^2 = g^2$ at the string scale M_X ,

$$\frac{g^2(M_X)}{4\pi} = \left(\frac{2^\beta}{n_g}\right)^{1/2} e^{\phi_4}, \quad (13)$$

with the value of ϕ_4 fixed by the value of the gauge couplings where they unify, $g^2(M_X)$, which assumes different values for models with different numbers of generations at $M_X = 2.2 \times 10^{16}$ GeV:

$$g^2|_{N_g=1}(M_X) = 0.275, \quad g^2|_{N_g=2}(M_X) = 0.358, \quad g^2|_{N_g=3}(M_X) = 0.511, \quad g^2|_{N_g=4}(M_X) = 0.895. \quad (14)$$

The corresponding string scale is then given by

$$M_{\text{St}} = \frac{g^2(M_X)}{4\pi} \left(\frac{n_g \pi}{2^\beta}\right)^{1/2} M_{\text{Planck}}, \quad (15)$$

where M_{Planck} is the reduced Planck scale, 2.44×10^{18} GeV.

After fixing the value of ϕ_4 , one can then determine the values of the gauge couplings for the hidden-sector gauge groups at the string scale:

$$g_{\text{USp}_j}^2 = 2^{(4-\beta/2)} \pi n_g^{(\rho_j/2)} e^{\phi_4}, \quad (16)$$

where $\rho_1 = \rho_2 = +1$ and $\rho_3 = \rho_4 = -1$. Using the beta-function parameters β_j in Table 2, the scale at which each hidden-sector gauge group becomes confining can be calculated:

$$\Lambda_j = M_X \cdot \exp \left\{ \frac{2\pi}{-\beta_j} \left[1 - \frac{2^\beta \pi}{g^2(M_X) n_g^{(\rho_j+1)/2}} \right] \right\}. \quad (17)$$

It can then be checked that the hidden-sector gauge groups have sufficiently negative β_j to become confining at high-energy scales. To have only one pair of light Higgs doublets, as is necessary in the MSSM in order for the gauge couplings to unify, one must fine-tune the mixing parameters of the Higgs doublets, specifically by fine-tuning the μ term in the superpotential, which may be generated via the higher-dimensional operators [33]:

$$W \supset \frac{y_\mu^{ijkl}}{M_{\text{St}}} S_L^i S_R^j H_u^k H_d^l, \quad (18)$$

where y_μ^{ijkl} are Yukawa couplings, M_{St} is the string scale, and the singlets S_R^j are assumed to receive string-scale VEVs, while the VEVs of the singlets S_L^i are TeV-scale. Note that this term may only be generated for the models with $n_g > 1$. The exact linear combinations that give the two light Higgs eigenstates are correlated with the pattern of Higgs VEVs necessary to obtain Yukawa matrices for the quarks and leptons,

$$H_{u,d} = \sum_i \frac{v_{u,d}^i}{\sqrt{\sum (v_{u,d}^i)^2}}, \quad (19)$$

where $v_{u,d}^i = \langle H_{u,d}^i \rangle$. Thus, at low energies one obtains MSSM-like models with different numbers of generations, with gauge-coupling unification $\sim 2.2 \times 10^{16}$ GeV, and matter charged under the hidden-sector gauge groups becomes confined into massive bound states at high-energy scales.

As has been mentioned, quantities such as gauge and Yukawa couplings depend on the VEVs of the closed-string moduli that parametrize the size and shape of the compactified manifold, as well as the open-string moduli that parametrize the positions of the D6-branes in the internal space, which are associated with the presence of three chiral adjoints in each stack. These VEVs should be determined dynamically. While it is not our goal to solve this problem in the present work, it should be mentioned that mechanisms do exist by which this can be accomplished. In particular, the closed-string moduli can be stabilized in AdS by turning on fluxes in Type IIA [53]. In fact, this mechanism has already been demonstrated for the three-generation model [54]. Also, gaugino condensation in the hidden sectors can provide another source of closed-string moduli stabilization [52]. The open-string moduli may be frozen if the D-branes wrap rigid cycles, a possibility that can exist on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with discrete torsion [55, 56, 57]. An example of a four-generation MSSM-like model constructed from D6-branes wrapping rigid cycles is given in [19]. We emphasize the possibility of finding a dynamical reason to explain why nature chooses a specific number of chiral generations by studying the moduli stabilization problem for models with different numbers of generations, such as our mini-landscape of models.

3. Yukawa Couplings. As one can see from the previous section (note the filler brane stacks in Table 3), only the models with $n_g \leq 4$ can satisfy the tadpole conditions without introducing fluxes. If we take this condition as a constraint, then the only viable models from the top-down point of view have $N_g = 1, 2, 3, 4, 6$, and 8. Furthermore, masses may be generated via trilinear

couplings for all generations only for those models with a tilted third torus ($\beta = 1$). If we also take this condition as a constraint, then the only viable models are those with $N_g = 1, 2, 3$, and 4. Additionally, the $SU(3)_C$ factor in the SM gauge group is only asymptotically free for SUSY models with four generations or less. Thus, the maximum viable number of generations is four.

The three-generation model has previously been studied in [33]. As mentioned in the Introduction, this model exhibits rank-3 Yukawa matrices and it is possible to nearly reproduce the correct masses and mixings for the three known generations of quarks and leptons. However, since a fourth generation has not yet been definitely ruled out, it is worth considering such models to see if they can reproduce the observed masses and mixings for the known quarks and leptons, while simultaneously satisfying experimental constraints on the fourth generation. In particular, the model with $n_g = 4$ and a tilted third torus ($\beta = 1$) has rank-4 Yukawa matrices, since all intersections occur on the first torus, similar to the three-generation model. In contrast to the three-generation model, the Yukawa matrices in the four-generation model may potentially be flavor democratic, and thus in some sense more natural. Therefore, studying the Yukawa textures generated by this model is of great interest.

A complete form for the Yukawa couplings y_{ij}^f for D6-branes wrapping on a full compact space $T^2 \times T^2 \times T^2$ can be expressed as [16, 32]:

$$Y_{\{ijk\}} = h_{qu} \sigma_{abc} \prod_{r=1}^3 \vartheta \left[\begin{matrix} \delta^{(r)} \\ \phi^{(r)} \end{matrix} \right] (\kappa^{(r)}), \quad (20)$$

where

$$\vartheta \left[\begin{matrix} \delta^{(r)} \\ \phi^{(r)} \end{matrix} \right] (\kappa^{(r)}) = \sum_{l \in \mathbf{Z}} e^{\pi i (\delta^{(r)} + l)^2 \kappa^{(r)}} e^{2\pi i (\delta^{(r)} + l) \phi^{(r)}}, \quad (21)$$

with $r = 1, 2, 3$ denoting the three two-tori. The input parameters are given by

$$\begin{aligned} \delta^{(r)} &= \frac{i^{(r)}}{I_{ab}^{(r)}} + \frac{j^{(r)}}{I_{ca}^{(r)}} + \frac{k^{(r)}}{I_{bc}^{(r)}} + \frac{d^{(r)}(I_{ab}^{(r)}\epsilon_c^{(r)} + I_{ca}^{(r)}\epsilon_b^{(r)} + I_{bc}^{(r)}\epsilon_a^{(r)})}{I_{ab}^{(r)}I_{bc}^{(r)}I_{ca}^{(r)}} + \frac{s^{(r)}}{d^{(r)}}, \\ \phi^{(r)} &= \frac{I_{bc}^{(r)}\theta_a^{(r)} + I_{ca}^{(r)}\theta_b^{(r)} + I_{ab}^{(r)}\theta_c^{(r)}}{d^{(r)}}, \\ \kappa^{(r)} &= \frac{J^{(r)}}{\alpha'} \frac{|I_{ab}^{(r)}I_{bc}^{(r)}I_{ca}^{(r)}|}{(d^{(r)})^2}. \end{aligned} \quad (22)$$

where the indices $i^{(r)}$, $j^{(r)}$, and $k^{(r)}$ label the intersections on the r^{th} torus, $d^{(r)} = \gcd(I_{ab}^{(r)}, I_{bc}^{(r)}, I_{ca}^{(r)})$, and the integer $s^{(r)}$ is a function of $i^{(r)}$, $j^{(r)}$, and $k^{(r)}$ corresponding to different ways of counting triplets of intersections. The shift parameters $\epsilon_a^{(r)}$, $\epsilon_b^{(r)}$, and $\epsilon_c^{(r)}$ correspond to the relative positions of stacks a , b , and c , while the parameters $\theta_a^{(r)}$, $\theta_b^{(r)}$, and $\theta_c^{(r)}$ are Wilson lines associated with these stacks. For simplicity, we set the Wilson lines to zero. The brane shifts and Wilson line together comprise the open-string moduli, which must be stabilized in a complete model, as mentioned previously. For the present work, we treat them as free parameters, as our primary interest is simply to see if one can obtain realistic mass matrices; we discussed possible mechanisms for moduli stabilization in the previous section. Note that although the above formulas for the Yukawa couplings are for $T^6 = T^2 \times T^2 \times T^2$, they may be extended to the present case $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ by including all of the orbifold images in the analysis. However, in the present case the cycles wrapped by the orbifold images of a stack of D-branes a are homologically identical

to the original cycle wrapped by the stack a . In addition, the intersection numbers between the cycles defined on the orbifold turn out to be the same as the intersection numbers between those on the ambient torus. Thus, the above formulas for $T^6 = T^2 \times T^2 \times T^2$ may be used without change on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$.

We focus only on the first torus, as the Yukawa couplings from the second and third tori only produce an overall constant. We label the left-handed fields, right-handed fields, and Higgs fields with the indices i , j , and k respectively, which may assume the values

$$i \in \{0, 1, 2, 3\}, \quad j \in \{0, 1, 2, 3\}, \quad k \in \{0, 1, 2, 3, 4, 5, 6, 7\}. \quad (23)$$

A trilinear Yukawa coupling occurs for a given set of indices that satisfy the selection rule

$$i + j + k = 0 \bmod 4. \quad (24)$$

The resulting rank-4 Yukawa matrices, for each of the U , D , and E sectors, have the form

$$Y \sim \sum_{j=0,4} \begin{pmatrix} Y_{00,j} \cdot v_j & Y_{01,3+j} \cdot v_{3+j} & Y_{02,2+j} \cdot v_{2+j} & Y_{03,1+j} \cdot v_{1+j} \\ Y_{10,3+j} \cdot v_{3+j} & Y_{11,2+j} \cdot v_{2+j} & Y_{12,1+j} \cdot v_{1+j} & Y_{13,j} \cdot v_j \\ Y_{20,2+j} \cdot v_{2+j} & Y_{21,1+j} \cdot v_{1+j} & Y_{22,j} \cdot v_j & Y_{23,3+j} \cdot v_{3+j} \\ Y_{30,1+j} \cdot v_{1+j} & Y_{31,j} \cdot v_j & Y_{32,3+j} \cdot v_{3+j} & Y_{33,2+j} \cdot v_{2+j} \end{pmatrix}, \quad (25)$$

where $v_k = \langle H_{k+1} \rangle$. These Yukawa matrices are rank 4, although they tend in practice to appear numerically close to being matrices of lower rank, due to both the periodic properties of Eq. (21) and the selection rule Eq. (24). FD Yukawa matrices can therefore arise even when the Higgs VEVs are all distinct and of the same order. However, the degree to which FD is broken depends upon differences between the Higgs VEVs as well as on the open-string and Kähler moduli, and so the detailed values of masses and mixings vary sensitively with the particular values of these parameters. Nevertheless, the moduli are not fine-tuned *per se*, since they turn out to be quite different for the U and D/E sectors. Note that the Yukawa matrices for the four-generation model with $n_g = 2, \beta = 0$ are at most rank 2, for then the intersection numbers on the first torus satisfy $\gcd(I_{ab}, I_{ac}, I_{ca}) = 2$. Thus, at most two generations can obtain distinct masses in that particular variant of the model.

4. Numerical Analysis. While the model described above supports rank-4 Yukawa matrices, the question of whether one can in fact obtain a phenomenologically suitable hierarchy of masses and mixings (which are somewhat different in the U , D , and E sectors) requires a detailed numerical study. In particular, it is of interest to see if the FD approach may be implemented. Strictly speaking, the observed masses and CKM elements must undergo RGE evolution to the unification scale M_{GUT} in order to be compared to model predictions. This requirement introduces two significant difficulties: First, the masses of the fourth-generation fermions, and their mixings with the other three, are of course unknown; for purpose of discussion, we simply take $m_{t'} = m_{b'} = m_{\tau'} = 400$ GeV, a choice that satisfies all current bounds [58]. Indeed, the combination of direct observation bounds, electroweak precision tests, and perturbative unitarity constraints from heavy-fermion scattering amplitudes place strong constraints on the possible masses, both lower and upper limits [59]. Second, a well-known problem of four-generation models, both supersymmetric and not, is the presence of several large Yukawa couplings (due to several fermions with electroweak-scale masses) that generate runaway Yukawa couplings at the TeV scale and above. While attempts have been made to stabilize the numerical evolution of the RGEs in

4-generation models up to M_{GUT} by including new matter fields (*e.g.*, [60]), questions of the robustness of such models remain. For our purposes, we simply assume that *some* supersymmetric model exists (see [61] for a related discussion) in which the Yukawa couplings all remain suitably small (from a perturbative viewpoint) to justify RGE evolution up to M_{GUT} , and take the values of the fourth-generation Yukawas at that scale to be 400 GeV divided by the mass of the corresponding third-generation fermion, while the Yukawas of the first three generations of fermions are those from the 3-generation MSSM RGE evolution to M_{GUT} [62, 63]. While such a fit is admittedly a hodgepodge from a phenomenological point of view, its purpose is merely to provide a proof of principle for the possibility of suitably hierarchical rank-4 Yukawas.

Specifically, since overall multiplicative factors in each Yukawa matrix Y_U , Y_D , Y_E remain undetermined [Eq. (20)], we fit to the fermion mass ratios $m_{k\ell}^{\text{ratio}} \equiv m_k/m_\ell$ in each sector and the independent hierarchical CKM elements $V_{ud} \sim V_{cs} \gg V_{us}$. To accommodate the hierarchical nature of the masses, we choose $\ln(m_{k\ell}^{\text{ratio,fit}}/m_{k\ell}^{\text{ratio,exp}})$ as the actual quantities to be fit, which we allow to vary by a chosen multiplicative factor (see below) in order to define a unit of χ^2 . Since the CKM elements are dimensionless, their analogous contributions to χ^2 are $\ln(|V_{ij}^{\text{fit}}/V_{ij}^{\text{exp}}|)$, so that the full χ^2 function reads:

$$\chi^2 = \sum_{k\ell} \left[\ln(m_{k\ell}^{\text{ratio,fit}}/m_{k\ell}^{\text{ratio,exp}}) / \ln(x_m) \right]^2 + \sum_{ij} \left[\ln(|V_{ij}^{\text{fit}}/V_{ij}^{\text{exp}}|) / \ln(x_{v,ij}) \right]^2, \quad (26)$$

where the x_m and $x_{v,ij}$ indicate that a multiplicative discrepancy by x in any observable amounts to a unit of χ^2 . We chose $x_m = 1.1$, *i.e.*, a 10% discrepancy, for mass ratios with indices in Eq. (26) ranging over $k\ell = tc, cu, tt', bs, sd, bb', \tau\mu, \mu e, \tau\tau'$, and $x_{v,ij} = 1.05$ for the diagonal $ij = ud, cs$ and $x_{v,ij} = 1.1$ for the off-diagonal $ij = us$ entries of the CKM matrix. Since all inputs are dimensionless, the fit is sensitive only to the ratios of VEVs $v_j^{(F)}/v_0^{(F)}$, where $j=1, \dots, 7$, and F refers to the appropriate Higgs VEV (that of H_U or H_D). The relevant model parameters all appear on the first torus; they are given by $\kappa' \equiv i\kappa^{(1)}$ and the shift parameters ϵ_m [*i.e.*, the portions of $\delta^{(1)}$ in Eq. (22) that are independent of i , j , and k], with $m = 1, 2, 3$ denoting the U, D, E sectors, respectively:

$$\epsilon_m \equiv \frac{1}{8} \left(\epsilon_{c,m}^{(1)} - \epsilon_{b,m}^{(1)} - 2\epsilon_{a,m}^{(1)} + 2s_m^{(1)} \right). \quad (27)$$

The numerical inputs and fit values of two sample fits are given in Table 4. A few comments on the fits are in order: First, we find that the minimization of χ^2 drives the simulation towards the vicinity of a stable minimum of the Kähler parameter $\kappa \sim 3$ even when starting far away from it, *e.g.*, $\kappa_{\text{init}} \sim 6$. Second, a similar stability of the minimum is observed for the VEVs $v_0^{(U),(D)}$. Third, we find that the VEVs obey the naturalness requirement of being all of the same order, *i.e.*, their ratios satisfy $v_j^{(U,D)}/v_0^{(U,D)} = O(1)$. Numerical simulations demonstrated the existence of a large number of solutions with parameters densely populating regions around typical ones shown in Table 4, with very small deviations from the computed values. These results fall roughly into two classes, with smaller and larger mixing between the third and fourth generation of quarks: Note the $V_{tb'}$ entries in Table 4. The first set of rows gives typical VEV ratios generated by the minimization of χ^2 . In the second set of rows, one sees that the size of χ^2 is driven by largely by discrepancies from the large fourth-generation fermion masses (~ 400 GeV), while the mass ratios of the SM fermions agree to a high level of accuracy. The mixing between the third and fourth generation is much smaller in the latter case. The values for extra

generations that we calculate from the model are consistent with recent fits to flavor physics data of Ref. [64] for $m_{t'} = 400\text{--}600$ GeV, $|V_{t'b'}| = 0.998 \pm 0.006$ and $|V_{t'b}| = 0.07 \pm 0.08$.

The resulting down-quark Yukawa parameters using the input parameters shown in the two columns on the left-hand side of Table 4 are

$$Y_D \sim \begin{pmatrix} 7.222 & 8.015 & 7.236 & 8.016 \\ 8.001 & 8.501 & 8.005 & 8.500 \\ 7.236 & 8.016 & 7.222 & 8.015 \\ 8.005 & 8.500 & 8.001 & 8.501 \end{pmatrix}, \quad (28)$$

while the up-quark Yukawa matrix is given by

$$Y_U \sim \begin{pmatrix} 5.868 & 14.371 & 5.837 & 14.415 \\ 10.969 & 5.495 & 10.966 & 5.498 \\ 5.837 & 14.415 & 5.868 & 14.371 \\ 10.966 & 5.498 & 10.969 & 5.495 \end{pmatrix}. \quad (29)$$

As can be seen, these matrices exhibit near degeneracies such that Y_U is close to rank 2, resulting in two heavy quarks (t' , t) and two light quarks (c , u). Y_D is also nearly rank 2, but (it turns out) is closer to rank 1 than Y_U , resulting in two heavy quarks (b' , b) and two lighter quarks (s , d), with b' one being significantly heavier than the others. As discussed earlier, these near-degeneracies arise naturally due to the selection rule Eq. (24) and the periodic properties of Eq. (21). However, it must be reiterated that particular numerical values of parameters marking the departure from FD are required to obtain these matrices, as seen from the choices of Higgs VEVs, the open-string moduli parameters $\epsilon_{a,b,c}$, and the Kähler modulus on the first torus. A complete model would of course provide a mechanism for stabilizing the moduli to these values. Alternately, some portion of the departure from FD Yukawa matrices may instead arise from D-brane instanton-induced couplings, rather than coming entirely from the Higgs and moduli VEVs.

The numerical analysis shows that the fourth-generation masses are naturally larger than those of the third generation. Additionally, we find a natural mass hierarchy between each generation. However, the model does not account for nonvanishing CKM elements between the second and third generation of fermions without significant deviations in mass ratios. The nontrivial values for these elements can easily be introduced by incorporating contributions to Yukawa matrices from four-point correlation functions [65]. Finally, we note that the above analysis provides a nice example of Flavor Democracy, which makes this analysis more natural and less fine-tuned compared to that of the three-generation model [33].

5. Conclusion. We have constructed a series of MSSM-like models with different numbers of chiral fermion generations from intersecting D6 branes on a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. Each of these models satisfies all global consistency conditions, including tadpole cancellation, K-theory constraints, and conditions for preserving $\mathcal{N} = 1$ supersymmetry. For each of the models, we also find that the tree-level gauge couplings are unified at the string scale. In addition, for the models constructed with one tilted two-torus, we find that the rank of the Yukawa mass matrices for quarks and leptons equals the number of generations. Thus, distinct masses for each generation and mixings between them may be generated.

The Yukawa mass matrices for the three-generation model have previously been studied. For this model it was found that it is possible to generate mass hierarchies and mixings that nearly

Table 4: Experimental input and sample fit values to the model. Note that we have fixed the values of the fourth-generation masses to be 400 GeV.

$\kappa' = 3.2452096, \quad \epsilon_1 = 0.203163427$ $\epsilon_2 = -0.090518048, \epsilon_3 = -0.090479810$ $\chi^2 = 111$			$\kappa' = 3.3338221, \quad \epsilon_1 = 0.210962338$ $\epsilon_2 = -0.078384580, \epsilon_3 = -0.079039780$ $\chi^2 = 142$		
j	$v_j^{(D)}/v_0^{(D)}$	$v_j^{(U)}/v_0^{(U)}$	j	$v_j^{(D)}/v_0^{(D)}$	$v_j^{(U)}/v_0^{(U)}$
1	1.1484243	1.2146412	1	1.1638620	1.1950213
2	0.9603816	0.7789428	2	0.9282357	0.7697829
3	1.2306571	1.4632390	3	1.2033923	1.1610830
4	1.2967893	0.7500008	4	1.3064203	0.7459922
5	1.1462177	2.6474058	5	1.1390428	1.2873315
6	1.4202164	0.8490424	6	1.4979862	0.8369313
7	1.2432859	2.2181503	7	1.2831043	1.2145519
$k\ell$	$m_{k\ell}^{\text{ratio,fit}}$	$m_{k\ell}^{\text{ratio,exp}}$	$k\ell$	$m_{k\ell}^{\text{ratio,fit}}$	$m_{k\ell}^{\text{ratio,exp}}$
tc	246.1185	247.50	tc	247.1696	247.50
cu	290.6168	290.60	cu	290.5549	290.60
$t't$	2.7551	2.2857	$t't$	5.4376	2.2857
bs	23.7721	35.700	bs	35.6756	35.700
sd	20.0002	19.860	sd	19.8532	19.860
$b'b$	93.7180	80.000	$b'b$	134.1494	80.000
$\tau\mu$	23.7466	21.830	$\tau\mu$	21.8330	21.830
μe	211.1299	211.10	μe	211.0802	211.10
$\tau'\tau$	93.7182	225.1	$\tau'\tau$	134.1025	225.1
ij	$ V_{ij}^{\text{fit}} $	$ V_{ij}^{\text{exp}} $	ij	$ V_{ij}^{\text{fit}} $	$ V_{ij}^{\text{exp}} $
ud	0.9763	0.9754	ud	0.9759	0.9754
us	0.2164	0.22050	us	0.2185	0.22050
cd	0.2164	0.22030	cd	0.2185	0.22030
tb	0.9815	0.9995	tb	0.9981	0.9995
tb'	0.1916	-	tb'	0.0610	-
$t'b'$	0.9815	-	$t'b'$	0.9981	-

match those that are observed [33]. In the present work, we studied the Yukawa matrices of the four-generation model and found that accomplishing the same for the known three generations while simultaneously satisfying constraints on fourth generation fermions may be possible. This conclusion, of course, comes with the caveat that the problem of the evolution of large Yukawa couplings can be ameliorated. Finally, the obtained Yukawa matrices provide a nice implementation of Flavor Democracy, and as such seem somewhat less fine-tuned in comparison to the three-generation model. In particular, the up-type Yukawa matrix is almost rank 2, resulting in two quarks with large masses and two with small masses. The down-type Yukawa matrix is nearly rank 1, resulting in one quark with a large mass, and three quarks with smaller masses.

As commented earlier, the only model of the type under study for which it is possible to generate masses and mixings for each generation are those in which the third two-torus is tilted. For this subset of models, the maximum number of generations that can be accommodated while simultaneously satisfying the tadpole constraints (without including supergravity fluxes) is four. Furthermore, it is known that the maximum number of generations in a supersymmetric model for which QCD is asymptotically free is also four.

Although disfavored, four-generation models are still presently viable phenomenologically. It is interesting that one can construct such a “realistic” model with four generations. At present, the reason that our universe seems to contain only three generations is unknown. Experimentally, the Large Hadron Collider (LHC) should be able to determine the question of the existence of the fourth generation definitively. From the top-down point-of-view, string theory does not yet appear to uniquely determine the number of generations. However, it may be possible to find dynamical reasons for singling out three generations once the moduli stabilization question has been completely addressed. We leave this question for future work.

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